ON BIFURCATION AND EXISTENCE
OF POSITIVE SOLUTIONS
FOR A CERTAIN $p$-LAPLACIAN SYSTEM

YIN XI HUANG AND JOSEPH W.-H. SO

1. Introduction. In this paper we study bifurcation of positive solutions for an elliptic system of the form

$$
\begin{cases}
-\Delta_p u_i + g_i(x,u_1,u_2) = \lambda_i |u_i|^{p-2} u_i & \text{in } \Omega \\
u_i = 0 & \text{on } \partial \Omega
\end{cases}
$$

$i = 1, 2$

on a smooth bounded domain $\Omega$ in $\mathbb{R}^N$, where $\Delta_p u = \text{div}(|\nabla u|^{p-2} \nabla u)$ is the $p$-Laplacian with $p > 1$. We will prove that under appropriate conditions on $g_i$, (1.1) has a continuum of positive solutions bifurcating from the trivial solution. In particular, it follows from our main result (Theorem 3.1) that the following competitive system

$$
\begin{cases}
-\Delta_p u_1 = |u_1|^{p-2} u_1 (\lambda_1 - a_{11} u_1 - a_{12} u_2) & \text{in } \Omega \\
u_1 = 0 & \text{on } \partial \Omega \\
-\Delta_p u_2 = |u_2|^{p-2} u_2 (\lambda_2 - a_{21} u_1 - a_{22} u_2) & \text{in } \Omega \\
u_2 = 0 & \text{on } \partial \Omega
\end{cases}
$$

admits positive solutions $(u_1, u_2)$, with $u_i > 0$, for some positive $\lambda_i$ and $a_{ij}, i, j = 1, 2$.

When $p = 2$, the $p$-Laplacian becomes the usual Laplacian and system (1.1) has been studied extensively. We refer to the work of Cantrell [5] and the reference therein. In the case when $p \neq 2$, $\Delta_p$ appears in numerous situations. For example, in the context of reaction-diffusions, Murray [16] suggested using diffusion of the form $\Delta_p u$ in the study of diffusion-kinetic enzymes problems. We mention [7] and [4] for other references. Recently, systems associated with the $p$-Laplacian have commanded growing interest. Fleckinger et al. [11, 12] studied the